Example For the given wave of: $E(z,t) = 5e^{-\alpha z} \sin(4\pi \times 10^9 t - 20\pi z)$

(a) find frequency, wavelength, and phase constant. If the medium of travel is non-magnetic, find ε_r.

(b) what is direction of travel

(c) if the wave is completely reflected back at $z=1\,m$, i.e. $\Gamma = 1$, what is the equation for the reflected wave

(d) At $z = 2m$, the amplitude of the wave is $1 \frac{V}{m}$. Find α.

Solution: (a) $\omega = 4\pi \times 10^9 \rightarrow 2\pi f = 4\pi \times 10^9 \rightarrow f = 2\times 10^9\ Hz = 2GHz$

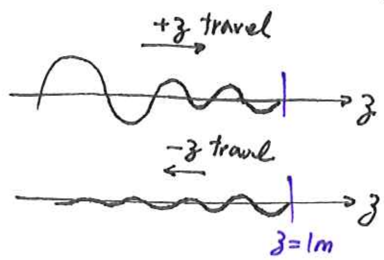
$$\beta = 20\pi \frac{rad}{m} \rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{20\pi} = \frac{1}{10}\ m$$

$$u_p = \frac{\omega}{\beta} = \frac{4\pi \times 10^9}{20\pi} = 2\times 10^8\ m/s$$

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} \rightarrow \varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3\times 10^8}{2\times 10^8}\right)^2 = 1.5^2 = 2.25$$

(b) The wave travels in $+z$ direction as the sign of t and z are different.

(c) At $z = 1m$ the amplitude is $5e^{-\alpha}$. Since the direction of travel is reversed, z changes sign $\Rightarrow$ $E_r(z,t) = 5e^{-\alpha} e^{\alpha z} \sin(4\pi \times 10^9 t + 20\pi z)$



(d) Amplitude $= 5e^{-\alpha z}\Big|_{z=2} = 1 \rightarrow 5e^{-2\alpha} = 1 \rightarrow e^{-2\alpha} = \frac{1}{5} \rightarrow -2\alpha = \ln\left(\frac{1}{5}\right) = -\ln 5$

$$\rightarrow \alpha = \frac{\ln 5}{2} = 0.81\ (m^{-1})$$

Example Find the instantaneous time sinusoidal functions corresponding to the following phasors:

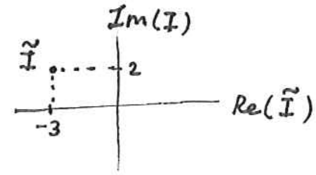
(a) $\tilde{V} = -5e^{j\pi/3}$ $\rightarrow \tilde{V} = 5e^{-j\pi}e^{j\pi/3} = 5e^{-j\frac{2\pi}{3}} \rightarrow V(t) = 5\cos(\omega t - \frac{2\pi}{3})$

(b) $\tilde{V} = j6e^{-j\pi/4}$ $\rightarrow \tilde{V} = e^{j\frac{\pi}{2}}\, 6e^{-j\frac{\pi}{4}} = 6e^{j\frac{\pi}{4}} \rightarrow V(t) = 6\cos(\omega t + \frac{\pi}{4})$

(c) $\tilde{I} = 6 + j8$ $\rightarrow \tilde{I} = \sqrt{6^2+8^2}\, e^{j\tan^{-1}\frac{8}{6}} = 10e^{j53.1^\circ} \rightarrow I(t) = 10\cos(\omega t + 53.1^\circ)$

(d) $\tilde{I} = -3 + j2$ $\rightarrow \tilde{I} = \sqrt{9+4}\, e^{j\tan^{-1}\frac{2}{-3}} = 3.61e^{j146.31^\circ} \rightarrow I(t) = 3.61\cos(\omega t + 146.31^\circ)$

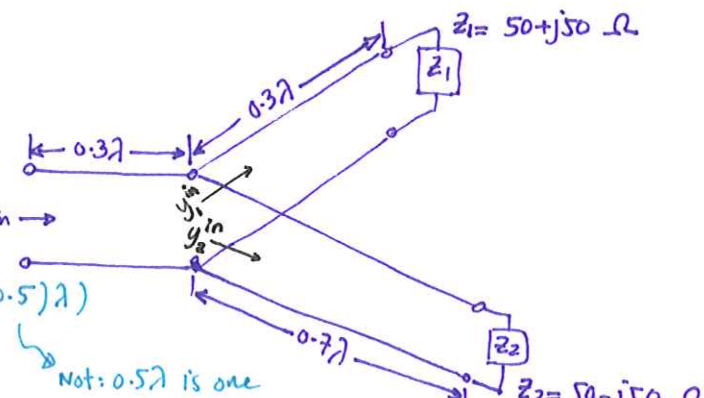
Note since $Real(\tilde{I}) = -3 < 0$ and $Im(\tilde{I}) = 2 > 0$, phase is between $\frac{\pi}{2}$ and π



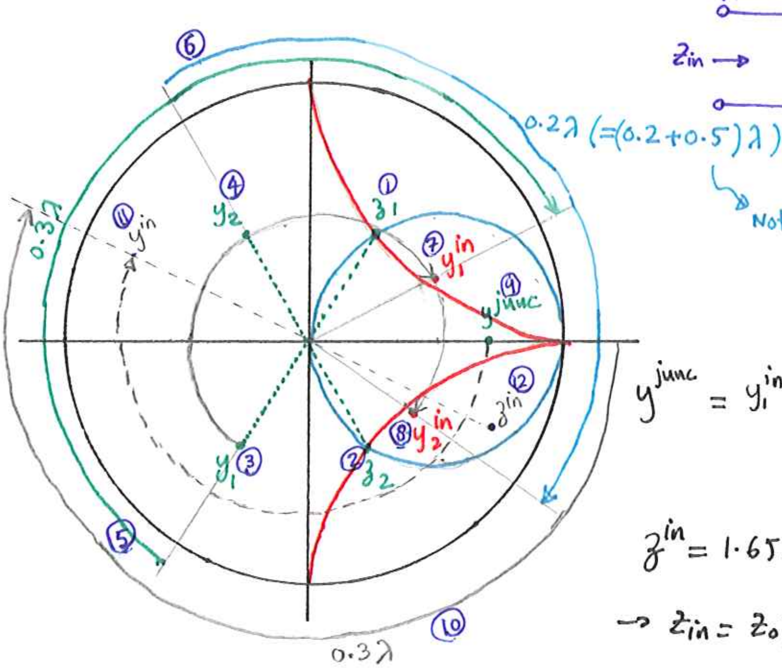
(e) $\tilde{I} = j$ $\rightarrow \tilde{I} = e^{j\frac{\pi}{2}} \rightarrow I(t) = \cos(\omega t + \frac{\pi}{2}) = -\sin\omega t$

(f) $\tilde{I} = 2e^{j\frac{\pi}{6}}$ $\rightarrow I(t) = 2\cos(\omega t + \frac{\pi}{6})$

Example use the smith chart to find Z_{in} of the feed line shown below. All lines are lossless with $Z_0 = 50\,\Omega$



Solution: $z_1 = \frac{Z_1}{50} = 1+j \qquad z_2 = \frac{Z_2}{50} = 1-j$



Not: 0.5λ is one complete turn. so we can extract that out.

$$y^{junc} = y_1^{in} + y_2^{in} = (1.97 + j1.02) + (1.97 - j1.02) = 3.94$$

$$z^{in} = 1.65 - j1.79$$

$$\rightarrow Z_{in} = Z_0 z^{in} = (50)(1.65 - j1.79) = 82.5 - j89.5\ \Omega$$

* Note: steps are indicated by numbers: ①, ②, ...

Example Given $\vec{A} = \hat{x}2 - \hat{y}3 + \hat{z}$ and $\vec{B} = \hat{x}B_x + \hat{y}2 + \hat{z}B_z$

(a) find B_x and B_z if $\vec{A}$ is parallel to $\vec{B}$

(b) find relation between B_x and B_z if $\vec{A}$ is perpendicular to $\vec{B}$

Solution: (a) $\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = 0 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -3 & 1 \\ B_x & 2 & B_z \end{vmatrix} = \hat{x}(-3B_z - 2) - \hat{y}(2B_z - B_x) + \hat{z}(4 + 3B_x) = 0$

$\rightarrow$ $-3B_z - 2 = 0 \rightarrow B_z = -\frac{2}{3}$

$2B_z - B_x = 0 \rightarrow 2(-\frac{2}{3}) - (-\frac{4}{3}) = 0$ ✓

$4 + 3B_x = 0 \rightarrow B_x = -\frac{4}{3}$

(b) $\vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0 = 2B_x - 6 + B_z = 0 \rightarrow B_z = 6 - 2B_x$

REVIEW of chapters 4 & 5

Maxwell's equations:

1) $\vec{\nabla} \cdot \vec{D} = \rho_v$

2) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

3) $\vec{\nabla} \cdot \vec{B} = 0$

4) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Electrostatics:

$\vec{\nabla} \cdot \vec{D} = \rho_v \rightarrow \int \vec{D} \cdot d\vec{s} = \int \rho_v \, dv = Q$

$\vec{\nabla} \times \vec{E} = 0 \rightarrow \oint \vec{E} \cdot d\vec{l} = 0$ Kirchhoff's Voltage law

Magnetostatics:

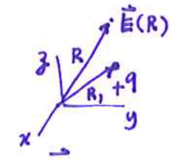
$\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \int \vec{B} \cdot d\vec{s} = 0$

$\vec{\nabla} \times \vec{H} = J \rightarrow \oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} = I$

charge density

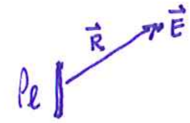
$$Q = \int_v \rho_v \, dv = \int_s \rho_s \, ds = \int_l \rho_l \, dl$$

Electric Field:

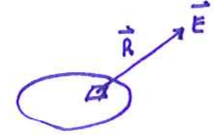


Point charge: $$\vec{E}_1 = \frac{q(\vec{R}-\vec{R}_1)}{4\pi\varepsilon|\vec{R}-\vec{R}_1|^3}$$ if Multiple charges: $$\vec{E} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i(\vec{R}-\vec{R}_i)}{|\vec{R}-\vec{R}_i|^3}$$

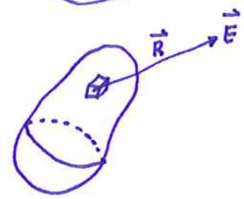
Line charge: $$\vec{E} = \frac{1}{4\pi\varepsilon}\int_l \hat{R}\,\frac{\rho_l\,dl}{R^2}$$



Surface charge: $$\vec{E} = \frac{1}{4\pi\varepsilon}\int_s \hat{R}\,\frac{\rho_s\,ds}{R^2}$$



Volume charge: $$\vec{E} = \frac{1}{4\pi\varepsilon}\int_V \hat{R}\,\frac{\rho_v\,dv}{R^2}$$



Electric Potential:

Point charge: $$V = \frac{q}{4\pi\varepsilon|R-R_1|}$$ if multiple charges: $$V = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i}{|\vec{R}-\vec{R}_i|}$$

Line charge: $$V = \frac{1}{4\pi\varepsilon}\int\frac{\rho_l\,dl}{R}$$

Surface charge: $$V = \frac{1}{4\pi\varepsilon}\int\frac{\rho_s\,ds}{R}$$

Volume charge: $$V = \frac{1}{4\pi\varepsilon}\int\frac{\rho_v\,dv}{R}$$

Poisson's Equation:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

If no free charge $\rightarrow$ Laplace's equation: $\nabla^2 V = 0$

Conductors:

* Drift velocity: $\vec{u}_e = -\underbrace{\mu_e}_{\text{electron mobility}}\vec{E}$ (m/s) $\qquad \vec{u}_h = \underbrace{\mu_h}_{\text{hole mobility}}\vec{E}$

* Current density: $\vec{J} = \rho_v\vec{u} = \rho_{ve}\vec{u}_e + \rho_{vh}\vec{u}_h = \vec{J}_e + \vec{J}_h$

* Conductivity: $\sigma = eN\mu = eN_e\mu_e + eN_h\mu_h$

Ohm's law: $\vec{J} = \sigma \vec{E}$ (point form) $\rightarrow R = \frac{V}{I} = \frac{1}{\sigma}\frac{\ell}{A}$ (Ω)

Resistance: $$R = \frac{V}{I} = \frac{-\int_\ell \vec{E}\cdot d\vec{\ell}}{\int_s \vec{J}\cdot d\vec{s}} = \frac{-\int_\ell \vec{E}\cdot d\vec{\ell}}{\int_s \sigma\vec{E}\cdot d\vec{s}}$$

Joule's law: $$P = \int_v \vec{E}\cdot\vec{J}\,dv = IV = RI^2 \quad (W)$$

Dielectrics

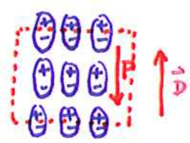
$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\overbrace{(1+\chi_e)}^{\varepsilon_r}\vec{E}$$

$\vec{P}$ ↳ Polarization field

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$

χ_e ↳ Electric susceptibility

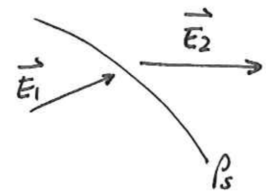
$$\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$$



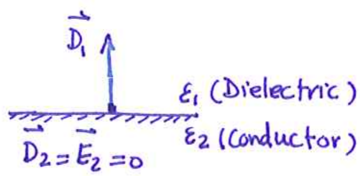
$\vec{E} = \frac{1}{\varepsilon_0}(\vec{D} - \vec{P})$

Electric Boundary Conditions

$$\begin{cases} E_{1t} = E_{2t} \\ D_{1n} - D_{2n} = \rho_s \end{cases}$$



Dielectric – Conductor Boundary:



Conductor – Conductor Boundary: $E_{1t} = E_{2t} \rightarrow \frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}$

$$J_{1n} = J_{2n} \;\&\; D_{1n} - D_{2n} = \rho_s \Rightarrow \varepsilon_1\frac{J_{1n}}{\sigma_1} - \varepsilon_2\frac{J_{1n}}{\sigma_2} = \rho_s$$

Capacitance

$$C = \frac{Q}{V} = \frac{\int_s \varepsilon\vec{E}\cdot d\vec{s}}{-\int_\ell \vec{E}\cdot d\vec{\ell}}$$

Parallel plate: $C = \varepsilon\frac{A}{d}$

$$RC = \frac{\varepsilon}{\sigma}$$

Electrostatic Potential Energy:

$$W = \frac{1}{2} C V^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\int_V \varepsilon E^2 \, dv \quad (J)$$

$\Rightarrow$ Force: $\vec{F} = -\vec{\nabla} W \quad (N)$

Chapter 5

Magnetic Force: $\vec{F}_m = q\vec{u}\times\vec{B} \quad (N)$ $\rightarrow$ Total Force: $F = q\vec{E} + q\vec{u}\times\vec{B}$ **Lorentz Force**

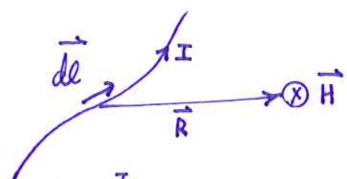
Force for a current carrying conductor: $\vec{F}_m = I\oint_C \vec{dl}\times\vec{B}$

Magnetic Torque: $\vec{T} = \vec{d}\times\vec{F} \quad (Nm)$

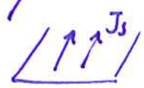
For a loop of N turns current: $\vec{T} = \vec{m}\times\vec{B} \quad (Nm)$

$$\vec{m} = N I A \hat{n} \quad (Am^2)$$

Biot_Savart law: $\vec{H} = \frac{I}{4\pi}\int_l \frac{\vec{dl}\times\hat{R}}{R^2} \quad \left(\frac{A}{m}\right)$



Surface current: $\vec{H} = \frac{1}{4\pi}\int_S \frac{\vec{J}_s\times\hat{R}}{R^2}\, ds$



Volume current: $\vec{H} = \frac{1}{4\pi}\int_V \frac{\vec{J}\times\hat{R}}{R^2}\, dv$

Amper's law: $\oint \vec{H}\cdot\vec{dl} = I$

Vector magnetic potential:

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\cdot\vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu\vec{J} \rightarrow \begin{cases} \nabla^2 A_x = -\mu J_x \\ \nabla^2 A_y = -\mu J_y \\ \nabla^2 A_z = -\mu J_z \end{cases}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_V \frac{\vec{J}}{R}\, dv \quad \left(\frac{Wb}{m}\right)$$

Magnetic flux Φ

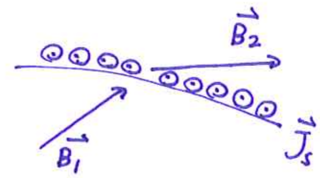
$$\Phi = \int_S \vec{B}\cdot d\vec{s} = \oint_C \vec{A}\cdot d\vec{l} \quad (\text{wb})$$

Magnetic Permeability

$$\vec{B} = \mu_0(\vec{H}+\vec{M}) = \mu_0(\vec{H}+\chi_m\vec{H}) = \mu_0\overbrace{(1+\chi_m)}^{\mu_r}\vec{H}$$

$$\vec{B} = \mu_0\mu_r\vec{H}$$

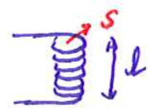
Magnetic Boundary Condition



$$\begin{cases} B_{1n} = B_{2n} \\ H_{2t} - H_{1t} = J_s \quad \left(\frac{A}{m}\right) \end{cases}$$

Self-inductance: $L \equiv \dfrac{\Lambda}{I}$ (H) $\quad \Lambda$: magnetic flux linkage $= N\int \vec{B}\cdot d\vec{s}$

For a solenoid: $L = \mu \dfrac{N^2}{l} S$



N turns.

Mutual inductance:

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2}{I_1}\int_{S_2} \vec{B_1}\cdot d\vec{s} \quad (\text{H})$$

Magnetic Energy: $W_m = \dfrac{1}{2}\displaystyle\int_V \mu H^2\, dv$